Solve 2nd order ODE using Frobenius' Method

1 $ay'' + by' + cy = 0$

$p(x) = \frac{b}{a}$, $q(x) = \frac{c}{a}$.

2 $y'' + p(x)y' + q(x)y = 0$

take denominator $= 0$

3 singular point/s x_0

- check $(x-x_0)p(x)$ and $(x-x_0)^2 q(x)$ analytic
- find $p_0 = \lim_{x \to x_0} (x-x_0)p(x)$, and $q_0 = \lim_{x \to x_0} (x-x_0)^2 q(x)$.

4 regular singular point/s x_0

substitute p_0 & q_0 into indicial equation $r(r-1) + p_0 r + q_0 = 0$

5 r_1 and r_2

choose largest r which r^*

6 ~~Proposed solution,~~ $w(r,x) = \sum_{n=0}^{\infty} a_n x^{n+r^*}$

derive w' and w''. substitute in ①

7 $aw'' + bw' + cw = 0$

rearrange according to power/index of x

8 $\sum_{n=0}^{\infty} \text{—} x^{\text{[illegible]}} + \sum_{n=0}^{\infty} \text{—} x^{\text{[illegible]}}$

shift indices

9

$$\boxed{\sim\sim + \sum_{n=\infty}^{\infty} \sim\sim\, x^{\boxed{}} = 0}$$

take $\sim\sim = 0$,

$\sim\sim = 0$,

10

$$\boxed{\begin{array}{c}\text{recurrence relation} \\ a_{next} = \sim\sim\, a_{previous}\end{array}}$$

express for few terms of a,

11

$$\boxed{a_{next} = \sim\sim\, a_0,}$$

substitute in proposed solution

12

$$\boxed{w(r^*, x) = a_0 x + \sim\sim\, a_0.}$$

express as a series.

13

$$\boxed{y_1(x) = \sum_{n=\sim}^{\infty} \sim\sim\, a_0 x^{n+r^*}}$$

Definition of Regular Singular Points (easier to prove)

Let $P(x)y'' + Q(x)y' + R(x)y = 0$,
be a linear second order differential equation.
Then x_0 is called a regular singular point if

$$\lim_{x \to x_0} (x - x_0) \frac{Q(x)}{P(x)} \quad \text{and} \quad \lim_{x \to x_0} (x - x_0)^2 \frac{R(x)}{P(x)}$$

are both finite.